Registration No.: | | | | | | | | | | |

Total Number of Pages: 02

Course: M. Sc.I.
Sub_Code: FMCC802

8th Semester Regular Examination: 2024-25
SUBJECT: Stochastic Process
BRANCH(S): M. Sc.I. (MC)
Time: 3 Hours
Max Marks: 70
Q.Code: S068
Answer Question No.1 (Part-I) which is compulsory, any five from rest (Part-II)
The figures in the right hand margin indicate marks.

Part-I

Q1 **Answer the following questions:** **(2 x 10)**

- **a)** Define Stochastic Process and give an example of it.
- **b)** How Markov chain is related to a stochastic process?
- **c)** Give examples of discrete time and continuous time Markov chain.
- **d)** What are transitional probabilities? Explain with an example.
- **e)** State Yule-Furry process.
- **f)** Explain Poisson process.
- **g)** Define Brownian Motion with an example.
- **h)** Define Renewal process.
- **i)** What is the difference between Renewal function and Renewal density?
- **j)** Write renewal and Wald's equations.

Part-II

Long Answer Type Questions (Answer Any five)

Q2 **a)** Let $\{X_n, n \geq 0\}$ be a Markov chain having state space $S = \{1, 2, 3\}$ and transition Matrix $P = \begin{bmatrix} 1/3 & 2/3 & 0 \\ 1 & 0 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$. Classify different types of states. **(5 + 5)**

b) Let a fair dice is tossed. Let the states of X_n be $k = 1, 2, \ldots\ldots, 6$, where k is the maximum number shown in n tosses. Find P and P^2.

Q3 **a)** Find P^n and the limiting probability vector V for the chain having transition probability matrix $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ p_1 & p_2 & p_3 \end{bmatrix}$, where $\sum p_i = 1$. **(5 + 5)**

b) Explain limiting behaviour of Markov chain. Also explain how finite irreducible Markov chain is related to Ergodic theorem.

Q4 **a)** Explain different types of state in Markov chain. **(5 + 5)**

b) Show that the interval between two successive occurrences of a Poisson process $\{N(t), t \geq 0\}$ having parameter λ has a negative exponential distribution with mean $\frac{1}{\lambda}$.

Q5 **a)** Explain Birth-Death process with an example. **(5 + 5)**

b) Write short notes on Weiner process.

Q6 **a)** Let $\{X(t), 0 \leq t \leq T\}$ be a Wiener process with $X(0) = 0$ and $\mu = 0$. Let $M(T)$ be the maximum of $X(t)$ in $0 \leq t \leq T$, *i.e.* $M(T) = \max_{0 \leq t \leq T} X(t)$. Then show that for any $a > 0$, $Pr\{M(T) \geq a\} = 2Pr\{X(T) \geq a\}$. **(5 + 5)**

b) What is stopping time in renewal process. Derive Wald's equation from it.

Q7 **a)** Show that the renewal function M satisfy the equation **(5 + 5)**

$$M(t) = F(t) + \int_0^t M(t-x)dF(x).$$

b) Differentiate between delayed and equilibrium renewal process by giving suitable examples.

Q8 **a)** State and prove Elementary Renewal theorem. **(5 + 5)**

b) The distribution of $N(t)$ is given by $p_n(t) = \Pr(N(t) = n) = F_n(t) - F_{n+1}(t)$ and the expected number of renewals are given by $M(t) = \sum_{n=1}^{\infty} F_n(t)$, where $N(t)$ is renewal process and $M(t)$ is renewal function.